

Problem 1.8

[Difficulty: 2]

1.8 Consider again the small particle of Problem 1.7. Express the distance required to reach 95 percent of its terminal speed in percent terms of g , k , and W .

Given: Small particle accelerating from rest in a fluid. Net weight is W ,
resisting force is $F_D = kV$, where V is speed.

Find: Distance required to reach 95 percent of terminal speed, V_t .

Solution: Consider the particle to be a system. Apply Newton's second law.

Basic equation: $\sum F_y = ma_y$

Assumptions:

1. W is net weight.
2. Resisting force acts opposite to V .

Then, $\sum F_y = W - kV = ma_y = m \frac{dV}{dt} = \frac{W}{g} V \frac{dV}{dy}$ or $1 - \frac{k}{W} V = \frac{V}{g} \frac{dV}{dy}$

At terminal speed, $a_y = 0$ and $V = V_t = \frac{W}{k}$. Then $1 - \frac{V}{V_t} = \frac{1}{g} V \frac{dV}{dy}$

Separating variables $\frac{V dV}{1 - \frac{V}{V_t}} = g dy$

Integrating, noting that velocity is zero initially

$$gy = \int_0^{0.95V_t} \frac{V dV}{1 - \frac{V}{V_t}} = \left[-VV_t - V_t^2 \ln \left(1 - \frac{V}{V_t} \right) \right]_0^{0.95V_t}$$

$$gy = -0.95V_t^2 - V_t^2 \ln(1 - 0.95) - V_t^2 \ln(1)$$

$$gy = -V_t^2 [0.95 + \ln 0.05] = 2.05 V_t^2$$

$$\therefore y = \frac{2.05}{g} V_t^2 = 2.05 \frac{W^2}{gt^2}$$

